

Review

of scientific supervisor, PhD, corresponding
member of the National Academy Science of the
Republic of Kazakhstan, Assoc. Prof.,
Durvudkhan Suragan on PhD thesis of Aidyn
Kassymov on theme
*"Basic functional and geometric inequalities
for the fractional order operators on
homogenous Lie groups"*

April 29, 2020

The PhD dissertation of Aidyn Kassymov "Basic functional and geometric inequalities for the fractional order operators on homogeneous Lie groups" is devoted to study fractional and geometric functional inequalities on homogeneous Lie groups.

This PhD dissertation has 4 main chapters investigating fractional functional inequalities on homogeneous Lie groups, and includes applications of these inequalities to nonlinear PDEs.

On Chapter 3, direct fractional functional inequalities on homogeneous Lie groups are given. In this chapter, fractional Hardy, Sobolev, Hardy-Sobolev, Gagliardo-Nirenberg, Caffarelli-Kohn-Nirenberg inequalities and their logarithmic versions on homogeneous Lie groups are obtained. For Riesz potential, Hardy-Littlewood-Sobolev and Stein-Weiss inequalities on homogeneous Lie groups are proved. The PhD candidate also establishes logarithmic Sobolev-Folland-Stein inequality on stratified Lie groups.

Chapter 4 is devoted to study reverse inequalities. Firstly, the PhD candidate obtained reverse integral of Hardy inequality on metric measure space with parameters $q < 0$, $p \in (0, 1)$ and $q \leq p < 0$. Consequently, the PhD candidate presents reverse integral of Hardy inequality on homogeneous Lie groups, hyperbolic space and Cartan-Hadamard manifolds. Also, he also proves reverse Hardy-Littlewood-Sobolev, Stein-Weiss, improved Stein-Weiss, Hardy,

L^p -Sobolev and L^p -Caffarelli-Kohn-Nirenberg inequalities on homogeneous Lie groups.

Chapter 5 is devoted to discuss applications of the obtained functional inequalities to nonlinear PDEs. The PhD candidate obtained Lyapunov inequality for the fractional p -sub-Laplacian on homogeneous Lie groups. Also, existence of weak solutions to the some nonlinear PDEs with nonlocal sources on different homogeneous Lie groups were obtained. In addition, the PhD candidate established blow-up results for the heat equations with fractional sub-Laplacian as well as logarithmic nonlinearity. Also, Fujita and Kato exponents are obtained for heat and wave Rockland equations on the graded Lie groups.

All the main results are published in peer-reviewed international scientific journals.

Considering significance and novelty of the obtained results, in my opinion, this PhD dissertation satisfies all the requirements and its author deserves the degree of the PhD in speciality "6D060100-Mathematics".



Scientific supervisor D. Suragan